University of Saida Dr Moulay Tahar
Faculty of M.I.T
Computer Science Department

Module: Operations Research
Promotion: 4th Year Engineer
Duration: 1h 30

FINAL EXAM

Exercise 1: [5 Points]

1. What does ILP mean and when should it be used?
2. Why do we need to use Binary Integer Linear Programming (Binary ILP)?
3. What is the key difference between CPM (MPM) and PERT?
4. What does Convex Programming mean?
5. What is the key concept of Gradient Descent and how does it work?
6. What do Heuristics and Metaheuristics mean?
7. What does Branch and Bound mean and what is its main purpose?
8. When do we need to use direct search methods instead of gradient-based methods?
9. What is the key difference between the Frank–Wolfe method and Kelley's Cutting Plane Method?
10. Explain the concept of feasible region in linear programming and its role in finding optimal solutions.

Exercise 2: [3 Points]

A research institution must allocate funding among six competing projects. The table below shows the funding requirement and impact score for each project:

Project	Funding ($1000s)	Impact Score
P1	25	12
P2	18	10
P3	22	14
P4	15	9
P5	30	16
P6	20	11

Constraints:

- Total funding available: $80,000
- Fund at most 4 projects
- If P5 is funded, P2 cannot be funded
- At least one of P1 or P3 must be funded
- Total high-risk projects ≤ 2 (P3 and P5 are high-risk)
- If P6 is funded, then P4 must also be funded

1. Define binary decision variables x_i for $i = 1, \ldots, 6$.
2. Formulate the complete binary ILP model.

Exercise 3: [6 Points]

A university plans to develop an online registration system for students. The project manager identified the activities listed in the table below.

Activity	Description	Duration	Predecessors
A	Requirements Analysis	3 days	—
B	System Design	4 days	A
C	Database Design	3 days	A
D	Frontend Development	5 days	B
E	Backend Development	6 days	B, C
F	System Integration	4 days	D, E
G	Testing and Validation	3 days	F
H	Deployment	2 days	G

1. Construct the **MPM network** (Activity-on-Node / Precedence Diagram).
2. Construct the **PERT network** (Activity-on-Arrow).
3. Identify and highlight the critical path on both networks. List all activities on the critical path.
4. Determine the total project duration and explain the significance of the critical path in project management.

Exercise 4: [6 Points]

Consider the following constrained optimization problem:

$$\begin{aligned} \text{minimize} \quad & f(x_1, x_2) = (x_1 - 3)^2 + (x_2 - 2)^2 \\ \text{subject to:} \quad & x_1 + x_2 \leq 4 \\ & x_1 \geq 0, \quad x_2 \geq 0 \end{aligned}$$

1. Show that the objective function is convex and compute its gradient $\nabla f(x_1, x_2)$.
2. Sketch the feasible region and identify its extreme points (vertices).
3. Apply the Frank–Wolfe algorithm starting from the initial feasible point $x^0 = (0, 0)$. Perform **two (2) full iterations** ($k = 0, 1$). For each iteration, clearly show:

- Compute the gradient $\nabla f(x^k)$
- Solve the linear subproblem (Linear Minimization Oracle): $s^k = \arg\min \langle \nabla f(x^k), s \rangle$ subject to $s \in$ feasible set
- Compute step size using the fixed rule: $\gamma_k = \frac{2}{k+2}$
- Update the iterate: $x^{k+1} = x^k + \gamma_k (s^k - x^k)$

4. Give the numerical values of x^1 and x^2 obtained after the iterations.
5. Briefly comment on whether the sequence is moving toward the unconstrained minimizer.